AP CALCULUS AB SYLLABUS

Sections Align to: Calculus for AP, 2nd Edition by R. Larson & P. Battaglia

MARKING PERIOD 1

Unit 1: Limits and Their Properties

- 1.2 Finding Limits Graphically and Numerically
- 1.3 Evaluating Limits Analytically
- 1.4 Continuity and One-Sided Limits
- 1.5 Infinite Limits
- 1.6 Limits at Infinity

Unit 2: Differentiation

- 2.1 The Derivative and the Tangent Line Problem
- 2.2 Basic Differentiation Rules and Rates of Change
- 2.3 Product and Quotient Rules, Trigonometric, and Higher-Order Derivatives
- 2.4 The Chain Rule and Exponential Functions
- 2.5 Implicit Differentiation and Logarithmic Differentiation
- 2.6 Derivatives of Inverse Functions
- 2.7 Related Rates

MARKING PERIOD 2

Unit 3: Applications of Differentiation

- 3.1 Extrema on an Interval
- 3.2 Rolle's Theorem and the Mean Value Theorem
- 3.3 Increasing and Decreasing Functions and the First Derivative Test
- 3.4 Concavity and the Second Derivative Test
- 3.5 A Summary of Curve Sketching
- 3.6 Optimization Problems
- 3.7 Linear Approximation and Differentials

Unit 4: Integration

- 4.1 Antiderivatives and Indefinite Integration
- 4.2 Area Under a Curve
- 4.3 Riemann Sums and Definite Integrals
- 4.4 The Fundamental Theorems of Calculus

Unit 4: Integration Continued

- 4.6 Integration by Substitution
- 4.7 The Natural Logarithmic Function: Integration
- 4.8 Inverse Trigonometric Functions: Integration

Unit 5: Differential Equations

- 5.1 Slope Fields
- 5.2 Growth and Decay
- 5.3 Separation of Variables

Unit 6: Applications of Integration

- 6.1 Area of Region Between Two Curves
- 6.2 Volume: The Disk and Washer Method

MARKING PERIOD 4

Unit 7: Integration Techniques, L'Hôpital's Rules, Partial Fractions

7.7 - Indeterminate Forms and L'Hôpital's Rule

REVIEW FOR AP TEST

7.4 - Integration by Parts

7.5 - Partial Fractions

Projects [as time allows]

ASSESSMENT INFORMATION

Marking Period 1	Marking Period 2	Marking Period 3	Marking Period 4
Major Summative (MAJ)	Major Summative (MAJ)	Major Summative (MAJ)	Major Summative (MAJ)
65%	65%	65%	65%
Minor Formative (MIN)	Minor Formative (MIN)	Minor Formative (MIN)	Minor Formative (MIN)
25%	25%	25%	25%
Class Participation (CP)	Class Participation (CP)	Class Participation (CP)	Class Participation (CP)
5%	5%	5%	5%
Homework (HW)	Homework (HW)	Homework (HW)	Homework (HW)
5%	5%	5%	5%

Black Horse Pike Regional School District Curriculum

ENGAGING STUDENTS • FOSTERING ACHIEVEMENT • CULTIVATING 21ST CENTURY GLOBAL SKILLS

Course Name: AP Calculus AB

Course Number: 0343000

PART I: UNIT RATIONALE - UNIT 1

WHY ARE STUDENTS LEARNING THIS CONTENT AND THESE SKILLS?

Unit 1 Title: Limits and Their Properties Grade Level(s): 12 AP Topics: 1.2, 1.3, 1.4, 1.5, 1.6, 1.7, 1.8, 1.9, 1.10, 1.11, 1.12, 1.13, 1.14, 1.15	Unit Summary: In this unit students develop an understanding of limits as the foundational building blocks for both derivatives and integration. It is essential for discovering and developing important ideas, definitions, formulas and theorems in calculus. Students will solve limit problems graphically, algebraically, and conceptually. They will generate and work with tables, sketch and analyze various graphs, and apply numerous algebraic techniques to find limits of indeterminate forms. Students must have a solid, intuitive understanding of limits and be able to compute various limits, such as, one- sided limits, limits at infinity, infinite limits, and trigonometric limits. In addition, they will communicate both orally and in written form effectively what their answers mean in the context of the problems they are given. Finally, students will understand how limits are used to determine continuity, which is a fundamental property of functions, and apply the Intermediate Value Theorem.
 Essential Question(s): What is a limit and how can you determine the limit of a function as x approaches c? What algebraic techniques can you use to evaluate a limit? What is continuity and how does it apply to the Intermediate Value Theorem? What is an infinite limit? 	 Enduring Understanding(s): Students will be able to: Represent limits analytically using correct notation Interpret limits expressed in analytic notation Estimate limits of functions Determine the limits of functions using limit theorems Determine the limits of functions using equivalent expressions for the function or the squeeze theorem. Justify conclusions about continuity at a point using the definition Determine intervals over which a function is continuous Determine values of <i>x</i> or solve for parameters that make discontinuous functions continuous, if possible Interpret the behavior of functions using limits involving infinity. Explain the behavior of a function on an interval using Intermediate Value Theorem

PART II: INSTRUCTIONAL STRATEGIES AND RESOURCES

DESCRIBE THE LEARNING TARGETS AND APPLICATIONS OF MATHEMATICAL PRACTICES FOR AP CALCULUS

AP College Board Mathematical Practices:

MPAC 1 – Implementing Mathematical Processes

Students will identify an appropriate mathematical rule or procedure based on the classification of a given expression such as using the chain rule to find the derivative of a composite function. They will apply appropriate mathematical rules or procedures, with or without technology. This will be achieved through discussion groups, sharing and responding, error analysis, distractor analysis, and model questions.

MPAC 2 – Connecting Representations

Students will identify mathematical information from graphical, numerica;, analytical, and/or verbal representation. They will identify a re-expression of mathematical information presented in a given representation and identify how mathematical characteristics or properties of functions are related in different representations. This will be achieved through creating representations, debriefing, rotation stations, and graphic organizers.

MPAC 3 – Justification

Students will identify an appropriate mathematical definition, theorem, or test to apply. They will confirm whether hypotheses or conditions of a selected definition, theorem, or test have been satisfied and apply an appropriate mathematical definition, theorem or test as support. Lastly, students will provide reasons or rationales for solutions and conclusions. This will be achieved through Think Alouds, critique reasoning, Error analysis, Whole group discussions and Think-Pair-Share

MPAC 4 – Communication and Notation

Students will use appropriate mathematical symbols and notation through match mine, model questions, and error analysis.

Real World and Interdisciplinary Problems:

Text: Calculus for AP 2nd Edition by R. Larson & P. Battaglia	Section: 1.2	Page : 74	
Description: Cost analysis functions for paddle board company			
Text: Calculus for AP 2nd Edition by R. Larson & P. Battaglia	Section: 1.3	Page : 86	
Description: Comparing velocity and position functions to make predictions			
Text: Calculus for AP 2nd Edition by R. Larson & P. Battaglia	Section: 1.4	Page : 91	
Description: Using Charles's Law and absolute value to determine a lower limit			
Description: Using Charles's Law and absolute value to determine a	ı lower limit		
Description: Using Charles's Law and absolute value to determine a Text: Calculus for AP 2nd Edition by R. Larson & P. Battaglia	lower limit Section: 1.5	Page : 107	
<i>Description:</i> Using Charles's Law and absolute value to determine a <i>Text:</i> Calculus for AP 2nd Edition by R. Larson & P. Battaglia <i>Description:</i> Utilizing average speed of vehicle between cities to interview.	<i>Section: 1.5</i> serpret limits	Page : 107	
<i>Description:</i> Using Charles's Law and absolute value to determine a <i>Text:</i> Calculus for AP 2nd Edition by R. Larson & P. Battaglia <i>Description:</i> Utilizing average speed of vehicle between cities to interference of the transmission of transmission of transmission of transmission of the transmission of trans	Section: 1.5 serpret limits Section: 1.6	Page: 107 Page: 117	

Students will Engage with the Following Text, Resources, and Tools:

Text:

Calculus for AP, 2nd Edition by Ron Larson and Paul Battaglia

Online Resources incorporated through the year, include but not limited to:

- Albert io Online AP Exam Practice Resource
- AP Central Previously Published and Released AP Questions
- CalcChat and CalcView Online Textbook Resource

Calculators:

- TI 84 Plus
- TI 89 Titanium

The Following 21st Century Skills and the 4 Mathematical Practices are Embedded Throughout the Course and are Evident in Daily Lessons, Assignments, Activities, Assessments, and Projects:

21 st Century skills:	NJ Standards for Mathematical Practices:
 Critical thinking Creativity Collaboration Communication Information literacy Technology literacy Media literacy Flexibility Leadership Initiative Productivity Social skills 	 Make sense of problems and persevere in solving them Reason abstractly and quantitatively Construct viable arguments and critique the reasoning of others. Model with mathematics Use appropriate tools strategically Attend to precision Look for and make use of structure Look for and express regularity in repeated reasoning

Students Will Write:

What the domain of a function is if given algebraically. What possible errors can be made by determining the domain of a function solely by analyzing a function's graph. The importance of examining a function analytically as well as graphically. A brief description of the meaning to the notation $\lim_{x\to e} f(x)$ Compare f(x) = x, g(x) = sinx, and $h(x) = \frac{sinx}{x}$ graphically and write why $\lim_{x\to 0} h(x) = 1$. Compare f(x) = x, g(x) = x, and $h(x) = \frac{x}{x}$ graphically and write why $\lim_{x\to 0} h(x) = 0$. About the importance of examining a function analytically as well as graphically when determining continuity. Descriptions about how functions differ. Meanings of different types of discontinuity and explain why. Explanations as to why the intermediate value theorem applies on a given closed interval.

PART III: TRANSFER OF KNOWLEDGE AND SKILLS

DESCRIBE THE LEARNING EXPERIENCE.

How Students Uncover Content and Build Skills:

Students will uncover and build skills through various classroom learning activities. Investigating algebra activities, modeling examples, using real-life application, using note-taking strategies, and using Smart Board technologies will all be explored as a blend of learning strategies to promote critical thinking, problem solving and performance skills of all learners. Other learning experiences could include alternative lesson openers, math and history applications, problem-solving workshops, interdisciplinary applications and projects.

Suggested warm-up activities, instructional strategies/activities, and assignments:

Section:1.2 – Finding Limits Graphically and Numerically

	Advanced Placement AB	
Focus and Motivate:	Suggestions include but not limited to:	
Starting Options (Lesson Warm Up)	Determine the vertical Asymptotes for the given functions	
Teaching Objectives	Represent limits analytically using correct	
	notation	
	 Interpret limits expressed ns in analytic notation 	
	Estimate limits of functions	
Checking for Understanding Suggestions include but not limited to:		
	Exit Tickets (teacher made supplement)	
	Inquiry	
	Formative Assessment	
Practice and Apply:	Larson Text Section: 1.2 Pg: 72 - 75	
Assigning Homework		

Section:1.3 – Evaluating Limits Analytically

	Advanced Placement AB	
Focus and Motivate: Starting Options (Lesson Warm Up)	Suggestions include but not limited to: Interpret the given limit and estimate its value	
Teaching Objectives	 Determine the limits of functions using limit theorems 	
	• Determine the limits of functions using equivalent expressions for the function or the squeeze theorem.	
Checking for Understanding	Suggestions include but not limited to: Exit Tickets (teacher made supplement) Inquiry Formative Assessment	
Practice and Apply: Assigning Homework	Larson Text Section: 1.3 Pg: 84 - 87	

Section:1.4 – Continuity and One-Sided Limits

	Advanced Placement AB	
Focus and Motivate: Starting Options (Lesson Warm Up)	Suggestions include but not limited to: Determine the value of the given limit and verify graphically using calculator	
Teaching Objectives	 Justify conclusions about continuity at a point using the definition Determine intervals over which a function is continuous Determine values of x or solve for parameters that make discontinuous functions continuous, if possible Explain the behavior of a function on an interval using the IVT 	
Checking for Understanding	Suggestions include but not limited to: Exit Tickets (teacher made supplement) Inquiry Formative Assessment	
Practice and Apply: Assigning Homework	Larson Text Section: 1.4 Pg: 96 - 99	

Section:1.5 – Infinite Limits

	Advanced Placement AB	
Focus and Motivate: Starting Options (Lesson Warm Up)	<i>Suggestions include but not limited to:</i> Define asymptote. What is a function's value at an asymptote? Explain.	
Teaching Objectives	 Interpret the behavior of functions using limits involving infinity 	
Checking for Understanding	Suggestions include but not limited to: Exit Tickets (teacher made supplement) Inquiry Formative Assessment	
Practice and Apply: Assigning Homework	Larson Text Section: 1.5 Pg: 105 - 107	

Section:1.6 – Limits at Infinity

	Advanced Placement AB
Focus and Motivate: Starting Options (Lesson Warm Up)	Suggestions include but not limited to: In your own words, describe the meaning of an infinite
Teaching Objectives	 Interpret the behavior of functions using limits involving infinity
Checking for Understanding	Suggestions include but not limited to: Exit Tickets (teacher made supplement) Inquiry Formative Assessment
Practice and Apply: Assigning Homework	Larson Text Section: 1.6 Pg: 115 - 117

PART IV: EVIDENCE OF LEARNING

IDENTIFY THE METHODS BY WHICH STUDENTS WILL DEMONSTRATE THEIR UNDERSTANDING OF CONTENT AND THEIR ABILITY TO APPLY SKILLS. IDENTIFY BLOOM'S LEVELS.



Formative Assessments:

The effectiveness of the instructional program will be based on numerous activities and strategies including the following: teacher observations, students collaborating with peers, questioning strategies, student record-keeping, quizzes, exit/admit assignments, peer/self-assessments, learning/response logs, discussions and practice presentations.

Accommodation(s)/Modification(s): As per Individual Students' IEP or 504 plan.

Summative Assessments:

The following assessments will be used to evaluate student learning, skill acquisition and academic achievement of the Standards of Mathematical Practice and the New Jersey Learning Standards for Mathematics listed under each chapter in the Algebra 1 curriculum/syllabus at the conclusion of an instructional time period.

- Diagnostic Pre-Test
- Chapter Tests
- Projects
- End-Of –Course Assessment

Accommodation(s)/Modification(s): As per Individual Students' IEP or 504 plan.

Performance Assessments:

The following assessments require students to utilize various strands of mathematics.

- Projects
- Practice AP Exam Questions
- Homework
- Classwork

PART I: UNIT RATIONALE - UNIT 2

WHY ARE STUDENTS LEARNING THIS CONTENT AND THESE SKILLS?

Unit 2 Title: Differentiation Grade Level(s): 12 AP Topics: 2.1, 2.2, 2.3, 2.4, 2.5, 2.6, 2.7, 2.8, 2.9, 2.10, 3.1, .32, 3.3, 3.4, 3.5, 3.6, 4.1, 4.2, 4.3, 4.4, 4.5	Unit Summary: In this unit students will get into the heart of calculus. Derivatives are a major portion of the course, so they will spend a significant amount of time in this unit. Derivatives are used to describe the rate of change of one variable with respect to another variable to understand change in a variety of contexts. At first students build the derivative using the concept of limits and use it primarily to compute the instantaneous rate of change of a function. Students should be able to use different definitions of the derivative, estimate derivatives from tables and graphs, and apply various derivative rules and properties. As they progress through this unit they will spend some time on the relationship between position, velocity and acceleration on problems involving projectile motion and rectilinear motion.
 Essential Question(s): What is a derivative and what is the relationship of continuity? How do you find the derivatives of basic algebraic functions, trigonometric functions, and exponential functions? How do you find the derivatives of functions involving products and quotients? How do you find the derivatives of composite functions, natural logarithmic functions, and exponential functions with bases other than e? How do you find the derivative of implicitly defined functions? How do you find the derivatives of inverse functions, including inverse trigonometric functions? What is a related rate and how do you find it? 	 Enduring Understanding(s): Students will be able to: Determine average rates of change using difference quotients Represent the derivative of a function as the limit of a difference quotient Determine the equation of a line tangent to a curve at a given point Estimate derivatives Interpret meaning of a derivative in context Explain relationship between differentiability and continuity Calculate derivatives of familiar functions Calculate rates of change in applied contexts Calculate derivatives of products and quotients of differentiable functions Determine higher-order derivatives of a function Calculate derivatives of compositions of differentiable functions Calculate derivatives of implicitly defined functions Calculate derivatives of inverse and inverse trigonometric functions Calculate related rates in applied contexts Interpret related rates in applied contexts

PART II: INSTRUCTIONAL STRATEGIES AND RESOURCES

DESCRIBE THE LEARNING TARGETS AND APPLICATIONS OF MATHEMATICAL PRACTICES FOR AP CALCULUS

Mathematical Practices:

MPAC 1 – Implementing Mathematical Processes

Students will identify an appropriate mathematical rule or procedure based on the classification of a given expression, and relationships between concepts to solve problems. They will apply appropriate mathematical rules or procedures, with and without technology. They will accomplish this through discussion groups, sharing and responding, error analysis, and scavenger hunts.

MPAC 2 – Connecting Representations

Students will identify common underlying structures in problems involving different contextual situations and identify mathematical information from graphical, numerical, analytical, and/or verbal representations. They will accomplish this through discussion groups, graphic organizers, rotation stations, and think-pair-share.

MPAC 3 – Justification

Students will apply an appropriate mathematical definition, teheore, or test. They will provide reasons or rationales for solutions and conclusions and explain the meaning of mathematical solutions in context. They will accomplish this through error analysis, model equations, scavenger hunts, graphic organizers and critique writing.

MPAC 4 – Communication and Notation

Students will appropriate mathematical symbols and notation. They will accomplish this through error analysis, answer clouds, and think-pair-share.

Real World and Interdisciplinary Problems:

Text: Calculus for AP 2nd Edition by R. Larson & P. Battaglia	Section: 2.1	Page : 124
Description: Tangent Line Problem - Optics and Refraction		
Text: Calculus for AP 2nd Edition by R. Larson & P. Battaglia	Section: 2.2	Page : 143
Description: Finding Velocity		
Text: Calculus for AP 2nd Edition by R. Larson & P. Battaglia	Section: 2.3	Page : 157
Description: Particle Motion		
Text: Calculus for AP 2nd Edition by R. Larson & P. Battaglia	Section: 2.4	Page : 171
Description: Doppler Effect		
Text: Calculus for AP 2nd Edition by R. Larson & P. Battaglia	Section: 2.5	Page : 181
Description: Trajectories (orthogonal)		
Text: Calculus for AP 2nd Edition by R. Larson & P. Battaglia	Section: 2.6	Page : 189
Description: Angular Speed of Light		
Text: Calculus for AP 2nd Edition by R. Larson & P. Battaglia	Section: 2.7	Page : 194
Description: Velocity of Piston		

Students will Engage with the Following Text, Resources, and Tools:

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Online Resources incorporated through the year, include but not limited to:

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The Following 21st Century Skills and the 4 Mathematical Practices are Embedded Throughout the Course and are Evident in Daily Lessons, Assignments, Activities, Assessments, and Projects:

21 st Century skills:	NJ Standards for Mathematical Practices:
 Critical thinking Creativity Collaboration Communication Information literacy Technology literacy Media literacy Flexibility Leadership Initiative Productivity Social skills 	 Make sense of problems and persevere in solving them Reason abstractly and quantitatively Construct viable arguments and critique the reasoning of others. Model with mathematics Use appropriate tools strategically Attend to precision Look for and make use of structure Look for and express regularity in repeated reasoning

Students Will Write:

Descriptions of the geometric significance of differentiability of two functions. The meaning of $\mathbb{C}'(\mathbb{P}, \mathbb{P})$ given $\mathbb{P}=\mathbb{C}(\mathbb{P})$ is the function that yields the number of gallons of gasoline sold by a station at a price of dollars/gal The criteria used in selecting a graph as \mathbb{P} and \mathbb{P}' when given two graphs.

Justifications as to why or why not a function has a tangent line at a given point.

The criteria for a function to have a horizontal tangent present.

Connections to differentiability and continuity.

Appropriate units with all answers that model real world situations - On the AP exam students may not earn a

point on a free-response question when units are not included with the solution.

Explanations how the velocity of an object can be determined given a graph of its position.

Describe the difference between the explicit form of a function and an implicit equation.

The difference between a negative rate of change and a positive rate of change.

Explain if x changes at a constant rate, does y change at a constant rate and if so is it the same as x.

Explain and justify their approach for solving related-rate problems.

PART III: TRANSFER OF KNOWLEDGE AND SKILLS

DESCRIBE THE LEARNING EXPERIENCE.

How Students Uncover Content and Build Skills:

Students will uncover and build skills through various classroom learning activities. Investigating algebra activities, modeling examples, using real-life application, using note-taking strategies, and using Smart Board technologies will all be explored as a blend of learning strategies to promote critical thinking, problem solving and performance skills of all learners. Other learning experiences could include alternative lesson openers, math and history applications, problem-solving workshops, interdisciplinary applications and projects.

Suggested warm-up activities, instructional strategies/activities, and assignments:

	Advanced Placement AB
Focus and Motivate: Starting Options (Lesson Warm Up)	Suggestions include but not limited to: Find the slope of the given secant line. Can you find slope with only 1 point?
Teaching Objectives	 Determine average rates of change using difference quotients Represent the derivative of a function as the limit of a difference quotient Determine the equation of a line tangent to a curve at a given point Estimate derivatives Interpret meaning of a derivative in context Explain relationship between differentiability and continuity Calculate derivatives of familiar functions
Checking for Understanding	Suggestions include but not limited to: Exit Tickets (teacher made supplement) Inquiry Formative Assessment
Practice and Apply: Assigning Homework	Larson Text Section: 2.1 Pg: 132-134

Section: 2.1 - The Derivative and The Tangent Line Problem

Section: 2.2 - Basic Differentiation Rules and Rates of Change

	Advanced Placement AB
Focus and Motivate: Starting Options (Lesson Warm Up)	Solve the following using first principles.
Teaching Objectives	 Calculate rates of change in applied contexts Interpret rates of change in applied contexts
Checking for Understanding	Suggestions include but not limited to: Exit Tickets (teacher made supplement) Inquiry Formative Assessment
Practice and Apply: Assigning Homework	Larson Text Section: 2.2 Pg: 144-147

	Advanced Placement AB
Focus and Motivate:	Suggestions include but not limited to:
Starting Options (Lesson Warm Up)	Find the equation of the tangent line
Teaching Objectives	 Calculate derivatives of products and quotients of differentiable functions Determine higher-order derivatives of a function
Checking for Understanding	Suggestions include but not limited to: Exit Tickets (teacher made supplement) Inquiry Formative Assessment
Practice and Apply: Assigning Homework	Larson Text Section: 2.3 Pg: 155-158

Section: 2.3 - Product & Quotient, Trigonometric, and Higher-Order Derivatives

Section: 2.4 - The Chain Rule and Exponential Functions

	Advanced Placement AB		
Focus and Motivate: Starting Options (Lesson Warm Up)	Suggestions include but not limited to: Determine f'(x) and dy/dx of the given functions		
Teaching Objectives	 Calculate derivatives of compositions of differentiable functions 		
Checking for Understanding	Suggestions include but not limited to: Exit Tickets (teacher made supplement) Inquiry Formative Assessment		
Practice and Apply: Assigning Homework	Larson Text Section: 2.4 Pg: 169-173		

Section: 2.5 - Implicit Differentiation and Logarithmic Differentiation

	Advanced Placement AB
Focus and Motivate: Starting Options (Lesson Warm Up)	Suggestions include but not limited to: Determine f'(x) of the given functions
Teaching Objectives	 Calculate derivatives of implicitly defined functions Justify conclusions about the behavior of an implicitly defined function based on evidence from its derivative
Checking for Understanding	Suggestions include but not limited to: Exit Tickets (teacher made supplement) Inquiry Formative Assessment
Practice and Apply: Assigning Homework	Larson Text Section: 2.5 Pg: 180-182

Section: 2.6 - Derivatives of Inverse Functions

	Advanced Placement AB
Focus and Motivate:	Suggestions include but not limited to:
Starting Options (Lesson Warm Up)	Solve for dy/dx. (use implicit differentiation)
Teaching Objectives	 Calculate derivatives of inverse and inverse
	trigonometric functions
Checking for Understanding	Suggestions include but not limited to: Exit Tickets (teacher made supplement) Inquiry Formative Assessment
Practice and Apply:	Larson Text Section: 2.6 Pg: 187-189
Assigning Homework	

Section: 2.7 - Related Rates

	Advanced Placement AB		
Focus and Motivate: Starting Options (Lesson Warm Up)	Suggestions include but not limited to: Determine if A, B, C, or D is the solution to the given problems (MC AP Practice)		
Teaching Objectives	 Calculate related rates in applied contexts Interpret related rates in applied contexts 		
Checking for Understanding	Suggestions include but not limited to: Exit Tickets (teacher made supplement) Inquiry Formative Assessment		
Practice and Apply: Assigning Homework	Larson Text Section: 2.7 Pg: 195-198		

PART IV: EVIDENCE OF LEARNING

IDENTIFY THE METHODS BY WHICH STUDENTS WILL DEMONSTRATE THEIR UNDERSTANDING OF CONTENT AND THEIR ABILITY TO APPLY SKILLS. IDENTIFY BLOOM'S LEVELS.



Formative Assessments:

The effectiveness of the instructional program will be based on numerous activities and strategies including the following: teacher observations, students collaborating with peers, questioning strategies, student record-keeping, quizzes, exit/admit assignments, peer/self-assessments, learning/response logs, discussions and practice presentations.

The following assessments will be used to evaluate student learning, skill acquisition and academic achievement of the Standards of Mathematical Practice and the New Jersey Learning Standards for Mathematics listed under each chapter in the Algebra 1 curriculum/syllabus at the conclusion of an instructional time period.

- Diagnostic Pre-Test
- Chapter Tests
- Projects
- End-Of Course Assessment

Accommodation(s)/Modification(s): As per Individual Students' IEP or 504 plan.

Performance Assessments:

The following assessments require students to utilize various strands of mathematics.

- Projects
- Practice AP Exam Questions
- Homework
- Classwork

Accommodation(s)/Modification(s): As per Individual Students' IEP or 504 plan.

PART I: UNIT RATIONALE - UNIT 3

WHY ARE STUDENTS LEARNING THIS CONTENT AND THESE SKILLS?

Unit 3 Title: Applications of Differentiation Grade Level(s): 12 AP Topics: 4.6, 5.1, 5.2, 5.3, 5.4, 5.5, 5.6, 5.7, 5.8, 5.8, 5.9, 5.10, 5.11, 5.12	Unit Summary: In this unit students will go on to understand and apply the Mean Value Theorem and will have the opportunity to see how the average rate of change can be used to justify instantaneous speed. They will also spend a significant amount of time developing a comprehensive analysis of functions (for example, determining whether a function is increasing or decreasing and finding concavity and extreme values), using not only their graphs but their derivatives as well. Students should be able to solve separable differential equations, and be familiar with a variety of real-world applications, including related rates, optimization, linear approximation and growth and decay models. This is most likely the first time students will be asked to think deeply on a conceptual level, so they may struggle to make connections at first. Students often ask how far they should simplify their answers and it should be stressed that they should look to simplify only if it provides meaningful progress, such as a much shorter or cleaner answer.
 Essential Question(s): What are extrema and how can you find them on open and closed intervals? What is the Mean Value Theorem and how is it used? How can you determine the intervals on which a function is increasing or decreasing and the location of the function's relative extrema? How do you determine the concavity of a function and find its inflection points? How do you analyze a function and sketch its graph? How do you maximize or minimize quantities? How are differentials used to explain the tangent line approximation? 	 Enduring Understanding(s): Students will be able to: Justify conclusions about functions by applying the Extreme Value Theorem Justify conclusions about the behavior of a function based on the behavior of its derivatives Determine critical points of implicit relations Justify conclusions about the behavior of a function based on the behavior of its derivatives Calculate minimum and maximum values in applied contexts or analysis of functions Interpret minimum and maximum values calculated in applied contexts Approximate a value on a curve using the equation of a tangent line

Mathematical Practices:

MPAC 1 – Implementing Mathematical Processes

Students will apply appropriate mathematical rules or procedures, with or without technology. They will achieve this through simplification techniques, think-pair-share, and distractor analysis

MPAC 2 – Connecting Representations

Students will identify how mathematical characteristics or properties of functions are related in different representations. They will also describe the relationships among different representations of functions and their derivatives. They will achieve this through error analysis, graphic organizers, representations, and quickwrite strategies.

MPAC 3 – Justification

Students will apply an appropriate mathematical definition, theorem, or test. They will provide reasons or rationales for solutions and clusions as well as the meaning of the solutions in appropriate contextual settings. They will accomplish this through error analysis, think alouds, critique writing, scavenger hunts, model questions, and graphic organizers.

MPAC 4 – Communication and Notation

Students will use appropriate units of measure and graphing techniques through creating representations, graph and switch, and error analysis.

Real World and Interdisciplinary Problems:

Text: Calculus for AP 2nd Edition by R. Larson & P. Battaglia	Section:	3.1	Page : 219
Description: lawn sprinklers and their trajectories			
Text: Calculus for AP 2nd Edition by R. Larson & P. Battaglia	Section:	3.2	Page : 223
Description: Finding instantaneous rate of change of cars			
Text: Calculus for AP 2nd Edition by R. Larson & P. Battaglia	Section:	3.3	Page : 226
Description: Why two times of flight plane has same speed			
Text: Calculus for AP 2nd Edition by R. Larson & P. Battaglia	Section:	3.4	Page : 243
Description: Highway design - Engineering			
Text: Calculus for AP 2nd Edition by R. Larson & P. Battaglia	Section:	3.5	Page : 243
Description: Graphical representation of 3D Geometric Properties			
Text: Calculus for AP 2nd Edition by R. Larson & P. Battaglia	Section:	3.6	Page : 262
Description: Farming plots			
Text: Calculus for AP 2nd Edition by R. Larson & P. Battaglia	Section:	3.7	Page : 269
Description: Error Estimation of Manufacturing products			

Students will Engage with the Following Text, Resources, and Tools:

Text:

Calculus for AP, 2nd Edition by Ron Larson and Paul Battaglia

Online Resources incorporated through the year, include but not limited to:

- Albert io online AP exam practice resource
- AP Central Previously published and released AP questions
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Calculators:

- TI 84 Plus
- TI 89 Titanium

The Following 21st Century Skills and the 4 Mathematical Practices are Embedded Throughout the Course and are Evident in Daily Lessons, Assignments, Activities, Assessments, and Projects:

21 st Century skills:	NJ Standards for Mathematical Practices:
 Critical thinking Creativity Collaboration Communication Information literacy Technology literacy Media literacy Flexibility Leadership Initiative Productivity Social skills 	 Make sense of problems and persevere in solving them Reason abstractly and quantitatively Construct viable arguments and critique the reasoning of others. Model with mathematics Use appropriate tools strategically Attend to precision Look for and make use of structure Look for and express regularity in repeated reasoning

Students Will Write:

How derivatives can be used to approximate the zero of a function. The difference between relative and absolute extrema.

The graph of a function on a closed interval given extrema locations.

Explain why or why not the Mean Value Theorem applies to a function on a closed interval.

Explain how the Mean Value Theorem and Rolle's theorem are similar and different.

How you can determine the intervals on which a function is increasing or decreasing.

How you can determine the location of a function's relative extrema using derivatives.

How to determine the concavity of a function using derivatives.

How can derivatives be used to sketch the graph of a function precisely.

Justify the possibility of a function crossing it's horizontal or vertical asymptotes.

To explain since the surface area of a bottle does not change when squeezed how does the volume change.

Justify how to maximize or minimize quantities and apply appropriate units.

PART III: TRANSFER OF KNOWLEDGE AND SKILLS

DESCRIBE THE LEARNING EXPERIENCE.

How Students Uncover Content and Build Skills:

Students will uncover and build skills through various classroom learning activities. Investigating algebra activities, modeling examples, using real-life application, using note-taking strategies, and using Smart Board technologies will all be explored as a blend of learning strategies to promote critical thinking, problem solving and performance skills of all learners. Other learning experiences could include alternative lesson openers, math and history applications, problem-solving workshops, interdisciplinary applications and projects.

Suggested warm-up activities, instructional strategies/activities, and assignments:

Section: 3.1 - Extrema on an Interval

	Advanced Placement AB
Focus and Motivate:	Suggestions include but not limited to:
Starting Options (Lesson Warm Up)	Find the coordinate of the vertices for the given functions
Teaching Objectives	 Justify conclusions about functions by applying the Extreme Value Theorem Justify conclusions about the behavior of a function based on the behavior of its derivatives Determine critical points of implicit relations
Checking for Understanding	Suggestions include but not limited to: Exit Tickets (teacher made supplement) Inquiry Formative Assessment
Practice and Apply: Assigning Homework	Larson Text Section: 3.1 Pg: 217-219

Section: 3.2 - Rolle's Theorem and the Mean Value Theorem

	Advanced Placement AB
Focus and Motivate:	Suggestions include but not limited to:
Starting Options (Lesson Warm Up)	Determine the max or min value of the function and determine if its a local or extrema
Teaching Objectives	 Justify conclusions about functions by applying the Mean Value Theorem over an interval
Checking for Understanding	Suggestions include but not limited to: Exit Tickets (teacher made supplement) Inquiry Formative Assessment
Practice and Apply: Assigning Homework	Larson Text Section: 3.2 Pg: 224-226

	Advanced Placement AB
Focus and Motivate: Starting Options (Lesson Warm Up)	Suggestions include but not limited to: Use the MVT to verify that an extrema point exists on the
	given interval
Teaching Objectives	 Justify conclusions about the behavior of a function based on the behavior of its derivatives
Checking for Understanding	Suggestions include but not limited to: Exit Tickets (teacher made supplement) Inquiry Formative Assessment
Practice and Apply: Assigning Homework	Larson Text Section: 3.3 Pg: 233-236

Section: 3.3 - Increasing and Decreasing Functions and the First Derivative Test

Section: 3.4 - Concavity and the Second Derivative Test

	Advanced Placement AB
Focus and Motivate:	Suggestions include but not limited to:
Starting Options (Lesson Warm Up)	Determine if the function is increasing or decreasing on the given interval. Verify by graphical analysis.
Teaching Objectives	 Justify conclusions about the behavior of a function based on the behavior of its derivatives
Checking for Understanding	Suggestions include but not limited to: Exit Tickets (teacher made supplement) Inquiry Formative Assessment
Practice and Apply: Assigning Homework	Larson Text Section: 3.4 Pg: 242-244

Section: 3.5 - A Summary of Curve Sketching

	Advanced Placement AB
Focus and Motivate:	Suggestions include but not limited to:
Starting Options (Lesson Warm Up)	What is concavity? Determine the intervals of concavity for the given graph.
Teaching Objectives	 Justify conclusions about the behavior of a function based on the behavior of its derivatives
Checking for Understanding	Suggestions include but not limited to: Exit Tickets (teacher made supplement) Inquiry Formative Assessment
Practice and Apply: Assigning Homework	Larson Text Section: 3.5 Pg: 253-256

Section: 3.6 - Optimization Problems

	Advanced Placement AB
Focus and Motivate: Starting Options (Lesson Warm Up)	Suggestions include but not limited to: Determine the intervals of increase and decrease and concavity given the following family of curve sketches.
Teaching Objectives	 Calculate minimum and maximum values in applied contexts or analysis of functions Interpret minimum and maximum values calculated in applied contexts
Checking for Understanding	Suggestions include but not limited to: Exit Tickets (teacher made supplement) Inquiry Formative Assessment
Practice and Apply: Assigning Homework	Larson Text Section: 3.6 Pg: 262-265

Section: 3.7 - Linear Approximation and Differentials

	Advanced Placement AB
Focus and Motivate: Starting Options (Lesson Warm Up)	Suggestions include but not limited to: Various optimization problems.
Teaching Objectives	 Approximate a value on a curve using the equation of a tangent line
Checking for Understanding	Suggestions include but not limited to: Exit Tickets (teacher made supplement) Inquiry Formative Assessment
Practice and Apply: Assigning Homework	Larson Text Section: 3.7 Pg: 272-275

PART IV: EVIDENCE OF LEARNING

IDENTIFY THE METHODS BY WHICH STUDENTS WILL DEMONSTRATE THEIR UNDERSTANDING OF CONTENT AND THEIR ABILITY TO APPLY SKILLS. IDENTIFY BLOOM'S LEVELS.



Formative Assessments:

The effectiveness of the instructional program will be based on numerous activities and strategies including the following: teacher observations, students collaborating with peers, questioning strategies, student record-keeping, quizzes, exit/admit assignments, peer/self-assessments, learning/response logs, discussions and practice presentations.

Summative Assessments:

The following assessments will be used to evaluate student learning, skill acquisition and academic achievement of the Standards of Mathematical Practice and the New Jersey Learning Standards for Mathematics listed under each chapter in the Algebra 1 curriculum/syllabus at the conclusion of an instructional time period.

- Diagnostic Pre-Test
- Chapter Tests
- Projects
- End-Of Course Assessment

Accommodation(s)/Modification(s): As per Individual Students' IEP or 504 plan.

Performance Assessments:

The following assessments require students to utilize various strands of mathematics.

- Projects
- Practice AP Exam Questions
- Homework
- Classwork

PART I: UNIT RATIONALE - UNIT 4

WHY ARE STUDENTS LEARNING THIS CONTENT AND THESE SKILLS?

Unit 4 Title: Integration Grade Level(s): 12 AP Topics: 6.1, 6.2, 6.3, 6.4, 6.5, 6.6, 6.7, 6.8, 6.9, 6.10, 6.14, 7.1, 7.6, 7.7, 8.1, 8.3	Unit Summary: In this unit students will discover the relationship between differentiation and integration as inverse operations. Students will learn how to integrate functions using definite integrals and indefinite integrals through the use of Riemann sums. Students will use integration to solve many real-world applications. Students will also learn the importance of the Fundamental Theorem of Calculus and its many applications. Students will revisit differentiation when they learn to integrate using transcendental functions. This will instill the importance of the chain rule which they learned in the Derivative unit.
 Essential Question(s): What are antiderivatives and how are they used? How can you approximate the area of a plane region? How are Riemann sums similar to the Trapezoidal Rule and how are they different? What is the Fundamental Theorem of Calculus? How do you integrate composite functions? How do you integrate rational functions and trigonometric functions other than sine or cosine? How can you recognize when an integral results in an inverse trigonometric function? 	 Enduring Understanding(s): Students will be able to: Evaluate definite integrals analytical using the Fundamental Theorem of Calculus Determine antiderivatives of functions and indefinite integrals, using knowledge of derivatives. Interpret verbal statements of problems as differential equations involving a derivative expression Determine general solutions to differential equations Determine particular solutions to differential equations Interpret the meaning of areas associated with the graph of a rate of change in context. Approximate a definite integral using geometric and numerical methods Interpret the limiting case of the Riemann sum as a definite integral Represent the limiting case of the Riemann sum as a definite integral Calculate a definite integral using areas and properties of definite integrals Interpret the meaning of a definite integral in accumulation problems. Calculate a definite integral using areas and properties of definite integrals Represent the average value of a function using definite integrals Represent accumulation functions using definite integrals Represent accumulation functions using definite integrals Represent accumulation functions using definite integrals Bertemine indefinite integrals b) Evaluate definite integrals

PART II: INSTRUCTIONAL STRATEGIES AND RESOURCES

DESCRIBE THE LEARNING TARGETS AND APPLICATIONS OF MATHEMATICAL PRACTICES FOR AP CALCULUS

Mathematical Practices:

MPAC 1 – Implementing Mathematical Processes

Students will identify an appropriate mathematical rule or procedure based on the classification of a given expression. They will identify an appropriate mathematical rule or procedure based on the relationship between concepts to solve problems, with and without technology. Lastly, they will explain how an approximated value relates to the actual value. This will be achieved through the use of discussion groups, sharing and responding, error analysis, model questions, and creating representations

MPAC 2 – Connecting Representations

Students will identify a re-expression of mathematical information presented in a given representation. They will identify how mathematical characteristics or properties of functions are related in different representations. This will be achieved through graph and switch, scavenger hunts, resting representations and quick writing.

MPAC 3 – Justification

Students will apply an appropriate mathematical definition, theorem, or test. This will be achieved through direct instruction, error analysis, graphic organizers, and reciprocated instruction through peer-to-peer discussions.

MPAC 4 – Communication and Notation

Students will use appropriate units of measure, mathematical symbols, and notation. This will be achieved through error analysis, think-pair-share, and marking texts.

Real World and Interdisciplinary Problems:

Text: Calculus for AP 2nd Edition by R. Larson & P. Battaglia	Section: 4.1 Page: 288
Description: Time-distance of grand canyon	
Text: Calculus for AP 2nd Edition by R. Larson & P. Battaglia	Section: 4.2 Page: 301
Description: Determine angle measures for manufacturing	
Text: Calculus for AP 2nd Edition by R. Larson & P. Battaglia	Section: 4.3 Page: 315
Description: Surveying of land with constraints	
Text: Calculus for AP 2nd Edition by R. Larson & P. Battaglia	Section: 4.4 Page: 322
Description: Calculating speed of sound	
Text: Calculus for AP 2nd Edition by R. Larson & P. Battaglia	Section: 4.6 Page: 345
Description: Maximum flow rates at pumping station	
Text: Calculus for AP 2nd Edition by R. Larson & P. Battaglia	Section: 4.7 Page: 355
Description: Heat transfers (Newton's Cooling Law)	
Text: Calculus for AP 2nd Edition by R. Larson & P. Battaglia	Section: 4.8 Page: 363
Description: Approximating value of Pi	

Students will Engage with the Following Text, Resources, and Tools:

Text:

• Calculus for AP, 2nd Edition by Ron Larson and Paul Battaglia

Online Resources incorporated through the year, include but not limited to:

- Albert io online AP exam practice resource
- AP Central Previously published and released AP questions
- CalcChat and Calc View Online Textbook Resource

Calculators:

- TI 84 Plus
- TI 89 Titanium

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21 st Century skills:	NJ Standards for Mathematical Practices:
 Critical thinking Creativity Collaboration Communication Information literacy Technology literacy Media literacy Flexibility Leadership Initiative Productivity Social skills 	 Make sense of problems and persevere in solving them Reason abstractly and quantitatively Construct viable arguments and critique the reasoning of others. Model with mathematics Use appropriate tools strategically Attend to precision Look for and make use of structure Look for and express regularity in repeated reasoning

Students Will Write:

Explain the difference, as you understand it so far, between definite integral and indefinite integral.

The difference between a differential and a derivative.

The difference between mean value theorem and the intermediate value theorem.

What are the different kinds of Riemann sums?

What evidence can you think of to show that Riemann sums really do get to the value of a definite integral found by the fundamental theorem as n approaches infinity.

PART III: TRANSFER OF KNOWLEDGE AND SKILLS

DESCRIBE THE LEARNING EXPERIENCE.

How Students Uncover Content and Build Skills:

Students will uncover and build skills through various classroom learning activities. Investigating algebra activities, modeling examples, using real-life application, using note-taking strategies, and using Smart Board technologies will all be explored as a blend of learning strategies to promote critical thinking, problem solving and performance skills of all learners. Other learning experiences could include alternative lesson openers, math and history applications, problem-solving workshops, interdisciplinary applications and projects.

Suggested warm-up activities, instructional strategies/activities, and assignments:

Section: 4.1 - Antiderivatives and Indefinite Integration

	Advanced Placement AB
Focus and Motivate: Starting Options (Lesson Warm Up)	Suggestions include but not limited to: Determine dy/dx for the following functions. Describe the process using words.
Teaching Objectives	 Evaluate definite integrals analytical using the Fundamental Theorem of Calculus Determine antiderivatives of functions and indefinite integrals, using knowledge of derivatives. Interpret verbal statements of problems as differential equations involving a derivative expression Determine general solutions to differential equations Determine particular solutions to differential equations
Checking for Understanding	Suggestions include but not limited to: Exit Tickets (teacher made supplement) Inquiry Formative Assessment
Practice and Apply: Assigning Homework	Larson Text Section: 4.1 Pg: 287-289

Section: 4.2 - Area Under a Curve

	Advanced Placement AB
Focus and Motivate:	Suggestions include but not limited to:
Starting Options (Lesson Warm Up)	Solve the following Integrals
Teaching Objectives	 Interpret the meaning of areas associated with the graph of a rate of change in context.

Checking for Understanding	Suggestions include but not limited to: Exit Tickets (teacher made supplement) Inquiry Formative Assessment
Practice and Apply:	Larson Text Section: 4.2 Pg: 299-301
Assigning Homework	

Section: 4.3 - Riemann Sums and Definite Integrals

	Advanced Placement AB
Focus and Motivate: Starting Options (Lesson Warm Up)	Suggestions include but not limited to: Using summation notation, determine the value of the given integral
Teaching Objectives	 Approximate a definite integral using geometric and numerical methods Interpret the limiting case of the Riemann sum as a definite integral Represent the limiting case of the Riemann sum as a definite integral Calculate a definite integral using areas and properties of definite integrals
Checking for Understanding	Suggestions include but not limited to: Exit Tickets (teacher made supplement) Inquiry Formative Assessment
Practice and Apply: Assigning Homework	Larson Text Section: 4.3 Pg: 312-315

Section: 4.4 - The Fundamental Theorem of Calculus

	Advanced Placement AB
Focus and Motivate:	Suggestions include but not limited to:
Starting Options (Lesson Warm Up)	Determine the area under the curve given an interval.
Teaching Objectives	 Determine the average value of a function using definite integrals Interpret the meaning of a definite integral in accumulation problems. Calculate a definite integral using areas and properties of definite integrals Represent accumulation functions using definite integrals
Checking for Understanding	Suggestions include but not limited to: Exit Tickets (teacher made supplement) Inquiry Formative Assessment
Practice and Apply: Assigning Homework	Larson Text Section: 4.4 Pg: 326-328

Section: 4.6 - Integration by Substitution

	Advanced Placement AB
Focus and Motivate: Starting Options (Lesson Warm Up)	Suggestions include but not limited to: Use the fundamental theorem of calculus to determine the average value of the function
Teaching Objectives	 For integrands requiring substitution or rearrangements into equivalent forms: a) Determine indefinite integrals b) Evaluate definite integrals
Checking for Understanding	Suggestions include but not limited to: Exit Tickets (teacher made supplement) Inquiry Formative Assessment
Practice and Apply: Assigning Homework	Larson Text Section: 4.6 Pg: 343-346

Section: 4.7 - The Natural Logarithmic Function: Integration

	Advanced Placement AB
Focus and Motivate: Starting Options (Lesson Warm Up)	Suggestions include but not limited to: Describe the differences between the first and second Fundamental Theorem of Calculus.
Teaching Objectives	 For integrands requiring substitution or rearrangements into equivalent forms: a) Determine indefinite integrals b) Evaluate definite integrals
Checking for Understanding	Suggestions include but not limited to: Exit Tickets (teacher made supplement) Inquiry Formative Assessment
Practice and Apply: Assigning Homework	Larson Text Section: 4.7 Pg: 353-355

Section: 4.8 - Inverse Trigonometric Functions: Integration

	Advanced Placement AB
Focus and Motivate:	Suggestions include but not limited to:
Starting Options (Lesson Warm Up)	Evaluate the given integral (logarithmic function)
Teaching Objectives	 For integrands requiring substitution or rearrangements into equivalent forms: a) Determine indefinite integrals b) Evaluate definite integrals
Checking for Understanding	Suggestions include but not limited to: Exit Tickets (teacher made supplement) Inquiry Formative Assessment
Practice and Apply: Assigning Homework	Larson Text Section: 4.8 Pg: 361-363

PART IV: EVIDENCE OF LEARNING

IDENTIFY THE METHODS BY WHICH STUDENTS WILL DEMONSTRATE THEIR UNDERSTANDING OF CONTENT AND THEIR ABILITY TO APPLY SKILLS. IDENTIFY BLOOM'S LEVELS.

Creating Evaluating Analyzing Applying Understanding Remembering

Formative Assessments:

The effectiveness of the instructional program will be based on numerous activities and strategies including the following: teacher observations, students collaborating with peers, questioning strategies, student record-keeping, quizzes, exit/admit assignments, peer/self-assessments, learning/response logs, discussions and practice presentations.

Accommodation(s)/Modification(s): As per Individual Students' IEP or 504 plan.

Summative Assessments:

The following assessments will be used to evaluate student learning, skill acquisition and academic achievement of the Standards of Mathematical Practice and the New Jersey Learning Standards for Mathematics listed under each chapter in the Algebra 1 curriculum/syllabus at the conclusion of an instructional time period.

- Diagnostic Pre-Test
- Chapter Tests
- Projects
- End-Of –Course Assessment

Accommodation(s)/Modification(s): As per Individual Students' IEP or 504 plan.

Performance Assessments:

The following assessments require students to utilize various strands of mathematics.

- Projects
- Practice AP Exam Questions
- Homework
- Classwork

PART I: UNIT RATIONALE - UNIT 5

WHY ARE STUDENTS LEARNING THIS CONTENT AND THESE SKILLS?

Unit 5 Title: Differential Equations Grade Level(s): 12 AP Topics: 7.1, 7.2, 7.3, 7.4, 7.6, 7.7, 7.8	Unit Summary: Students will see in this unit a substantial amount of real-life applications. It starts with the process of differential equations with both general and particular solutions, with an emphasis on slope fields and how they produce solutions. It is followed up with specific growth and decay models that are tied to everyday situations. Separation of variables follows with more emphasis placed on real-world applications including but not limited to: population growth, financial mathematics, and other scientific principles such as radioactive decay.
 Essential Question(s): How do you approximate the particular solution of a differential equation? How are differential equations used in application problems, such as the exponential growth and decay model? How do you solve separable differential equations? 	 Enduring Understanding(s): Students will be able to: Interpret verbal statements of problems as differential equations involving a derivative expression. Verify solutions to differential equations. Estimate solutions to differential equations. Determine general solutions to differential equations. Determine particular solutions to differential equations. Interpret the meaning of a differential equation and its variables in context. Determine general and particular solutions for problems involving differential equations in context.

PART II: INSTRUCTIONAL STRATEGIES AND RESOURCES

DESCRIBE THE LEARNING TARGETS AND APPLICATIONS OF MATHEMATICAL PRACTICES FOR AP CALCULUS

Mathematical Practices:

MPAC 1 – Implementing Mathematical Processes

Students will apply appropriate mathematical rules or procedures, with and without technology. In both slope field and differential equation activities, students learn the importance of correctly using the rules of algebra to separate the variable and then integrating using good mathematics. They will accomplish this through pair-share techniques, discussion groups and solving error analysis problems.

MPAC 2 – Connecting Representations

Students will identify a re-expression of mathematical information presented in a given representation. When defining e students will learn how to interpret various nomenclatures and to correctly differentiate and integrate the different transcendental functions. Students need to be comfortable using proper notation with differential equations and then solving for their final answer in proper slope-intercept format. They will complete this through structured warm-up problems, discussion groups and graphic organizers.

MPAC 3 – Justification

Students will confirm that solutions are accurate and appropriate. Students need to correctly use units in their final answers and explain their final answers in a contextual setting: i.e. for area problems units should be squared feet, and volume problems units should be feet cubed. They will accomplish this through checking their work and doing pair-share exercises.

MPAC 4 – Communication and Notation

Students will use appropriate graphing techniques. They will use a graphing calculator to confirm their results and other online resources available to them. Students learn how to interpret the results from their graphing calculators for solving problems where answers need precision. Students need to explain the relationship between differential equations and the function, as well as communicating the results they get from a slope field.

Real World and Interdisciplinary Problems:

Text: Calculus for AP 2nd Edition by R. Larson & P. Battaglia	Section: 5.1	Page : 377	
Description: Temperature - finding the general solution and co	omparing it to the exact s	olution.	
Text: Calculus for AP 2nd Edition by R. Larson & P. Battaglia	Section: 5.2	Page : 382	
Description: Using an exponential growth model to estimate p	opulation growth over ti	me.	
Text: Calculus for AP 2nd Edition by R. Larson & P. Battaglia	Section : 5.3	Page : 390	
Description: Modeling advertising awareness using separation	of variables.		

Students will Engage with the Following Text, Resources, and Tools:

Text:

Calculus for AP, 2nd Edition by Ron Larson and Paul Battaglia

Online Resources incorporated through the year, include but not limited to:

- Albert io online AP exam practice resource
- AP Central Previously published and released AP questions
- CalcChat and Calc View Online Textbook Resource

Calculators:

- TI 84 Plus
- TI 89 Titanium

The Following 21st Century Skills and the 4 Mathematical Practices are Embedded Throughout the Course and are Evident in Daily Lessons, Assignments, Activities, Assessments, and Projects:

21 st Century skills:	NJ Standards for Mathematical Practices:
 Critical thinking Creativity Collaboration Communication Information literacy Technology literacy Media literacy Flexibility Leadership Initiative Productivity Social skills 	 Make sense of problems and persevere in solving them Reason abstractly and quantitatively Construct viable arguments and critique the reasoning of others. Model with mathematics Use appropriate tools strategically Attend to precision Look for and make use of structure Look for and express regularity in repeated reasoning

Students Will Write:

Explain the difference between a differential and a derivative.

Explain how slope fields and numerical methods can be used to solve differential equations without finding an algebraic solution.

Describe the difference between a particular solution and a general solution.

Explain the difference between growth and decay.

PART III: TRANSFER OF KNOWLEDGE AND SKILLS

DESCRIBE THE LEARNING EXPERIENCE.

How Students Uncover Content and Build Skills:

Students will uncover and build skills through various classroom learning activities. Investigating algebra activities, modeling examples, using real-life application, using note-taking strategies, and using Smart Board technologies will all be explored as a blend of learning strategies to promote critical thinking, problem solving and performance skills of all learners. Other learning experiences could include alternative lesson openers, math and history applications, problem-solving workshops, interdisciplinary applications and projects.

Suggested warm-up activities, instructional strategies/activities, and assignments:

Section: 5.1 – Slope Fields

	Advanced Placement AB
Focus and Motivate:	Suggestions include but not limited to:
Starting Options (Lesson Warm Up)	Discuss the idea of slope and the difference how positive,
	negative, zero and undefined slopes are represented on a
	graph.

Teaching Objectives	 Interpret verbal statements of problems as differential equations involving a derivative expression. Verify solutions to differential equations. Estimate solutions to differential equations.
Checking for Understanding	Suggestions include but not limited to: Exit Tickets (teacher made supplement) Inquiry Formative Assessment
Practice and Apply: Assigning Homework	Larson Text Section: 5.1 Pg: 375-377

Section: 5.2 – Growth and Decay

	Advanced Placement AB
Focus and Motivate: Starting Options (Lesson Warm Up)	Suggestions include but not limited to: Discuss situations real world applications in medicine and science which require the modeling of growth and decay.
Teaching Objectives	 Determine general and particular solutions for problems involving differential equations in context. Interpret the meaning of a differential equation and its variables in context.
Checking for Understanding	Suggestions include but not limited to: Exit Tickets (teacher made supplement) Inquiry Formative Assessment
Practice and Apply: Assigning Homework	Larson Text Section: 5.2 Pg: 384-386

Section: 5.3 – Separation of Variables

	Advanced Placement AB
Focus and Motivate: Starting Options (Lesson Warm Up)	Suggestions include but not limited to: Find the integral of a transcendental function using u- substitution.
Teaching Objectives	 Determine general solutions to differential equations. Determine particular solutions to differential equations.
Checking for Understanding	Suggestions include but not limited to: Exit Tickets (teacher made supplement) Inquiry Formative Assessment
Practice and Apply: Assigning Homework	Larson Text Section: 5.3 Pg: 393-396

PART IV: EVIDENCE OF LEARNING

IDENTIFY THE METHODS BY WHICH STUDENTS WILL DEMONSTRATE THEIR UNDERSTANDING OF CONTENT AND THEIR ABILITY TO APPLY SKILLS. IDENTIFY BLOOM'S LEVELS.



Formative Assessments:

The effectiveness of the instructional program will be based on numerous activities and strategies including the following: teacher observations, students collaborating with peers, questioning strategies, student record-keeping, quizzes, exit/admit assignments, peer/self-assessments, learning/response logs, discussions and practice presentations.

Accommodation(s)/Modification(s): As per Individual Students' IEP or 504 plan.

Summative Assessments:

The following assessments will be used to evaluate student learning, skill acquisition and academic achievement of the Standards of Mathematical Practice and the New Jersey Learning Standards for Mathematics listed under each chapter in the Algebra 1 curriculum/syllabus at the conclusion of an instructional time period.

- Diagnostic Pre-Test
- Chapter Tests
- Projects
- End-Of –Course Assessment

Accommodation(s)/Modification(s): As per Individual Students' IEP or 504 plan.

Performance Assessments:

The following assessments require students to utilize various strands of mathematics.

- Projects
- Practice AP Exam Questions
- Homework
- Classwork

PART I: UNIT RATIONALE - UNIT 6

WHY ARE STUDENTS LEARNING THIS CONTENT AND THESE SKILLS?

Unit 6 Title: Applications of Integration Grade Level(s): 12 AP Topics: 8.1, 8.2, 8.3, 8.4, 8.5, 8.6, 8.7, 8.8, 8.9, 8.10, 8.11, 8.12	Unit Summary: Students will use integration to find the area between two curves. They will also use rotation and other techniques to find the volume of a variety of 3-D solids as well as of solids with known cross-sectional areas. The First Fundamental Theorem of Calculus will play a major role in the development in understanding of both area and volume, and students will have to rely on a variety of integration techniques to help them progress through the chapter including: substitution, transcendental and trigonometric techniques.
 Essential Question(s): How do you find the area of a region between two curves? How can you use integrals to find the volume of a solid? 	 Enduring Understanding(s): Students will be able to: Calculate areas in the plan using the definite integral. Calculate volumes of solids with known cross sections using definite integrals. Calculate volumes of solids of revolution using definite integrals.

PART II: INSTRUCTIONAL STRATEGIES AND RESOURCES

DESCRIBE THE LEARNING TARGETS AND APPLICATIONS OF MATHEMATICAL PRACTICES FOR AP CALCULUS

Mathematical Practices:

MPAC 1 – Implementing Mathematical Processes

Students will identify an appropriate mathematical rule or procedure based on the relationship between concepts or processes and they will apply appropriate mathematical rules or procedures, with and without technology. This will be achieved through direct instruction, pair-share activities and technology activities with the graphing calculator.

MPAC 2 – Connecting Representations

Students will identify mathematical information from graphical, symbolic, numerical, and/or verbal representations. They will also identify how mathematical characteristics or properties of functions are related in different representations. They will accomplish these through discussion groups, group activities and activities with functions using multiple representations.

MPAC 3 – Justification

Students will apply an appropriate mathematical definition, theorem or test. They will provide reasons or rationales for solutions and conclusions and explain the meaning of mathematical solutions in context. They will accomplish this through error analysis, model equations, graphic organizers and critique writing.

MPAC 4 – Communication and Notation

Students will use appropriate mathematical symbols and notations including proper indefinite and definite integral notation.

Real World and Interdisciplinary Problems:

Text: Calculus for AP 2nd Edition by R. Larson & P. Battaglia	Section: 6.1	Page : 418	
Description: Building Design using Area between 2 curves and volume of regularly shaped objects.			
Text: Calculus for AP 2nd Edition by R. Larson & P. BattagliaSection: 6.2Page: 429			
Description: 3-D Printing using the volume rotated around the y-axis.			
Text: Calculus for AP 2nd Edition by R. Larson & P. Battaglia	Section: 6.2	Page : 429	
Description: Volume of a fuel tank rotated around the x-axis.			

Students will Engage with the Following Text, Resources, and Tools:

Text:

• Calculus for AP, 2nd Edition by Ron Larson and Paul Battaglia

Online Resources incorporated through the year, include but not limited to:

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21 st Century skills:	NJ Standards for Mathematical Practices:
 Critical thinking Creativity Collaboration Communication Information literacy Technology literacy Media literacy Flexibility Leadership Initiative Productivity Social skills 	 Make sense of problems and persevere in solving them Reason abstractly and quantitatively Construct viable arguments and critique the reasoning of others. Model with mathematics Use appropriate tools strategically Attend to precision Look for and make use of structure Look for and express regularity in repeated reasoning

Students Will Write:

Explain the difference between finding area and volume.

Write how to find an area of a region using cross sections. How is this different from rotations?

Explain the difference between finding the volume of a disk and a washer. How are they represented differently in calculus?

Explain how finding area and volume can be used in a real-world situation. How can this be applied to finding volumes of irregularly shaped objects?

PART III: TRANSFER OF KNOWLEDGE AND SKILLS

DESCRIBE THE LEARNING EXPERIENCE.

How Students Uncover Content and Build Skills:

Students will uncover and build skills through various classroom learning activities. Investigating algebra activities, modeling examples, using real-life application, using note-taking strategies, and using Smart Board technologies will all be explored as a blend of learning strategies to promote critical thinking, problem solving and performance skills of all learners. Other learning experiences could include alternative lesson openers, math and history applications, problem-solving workshops, interdisciplinary applications and projects.

Suggested warm-up activities, instructional strategies/activities, and assignments:

	Advanced Placement AB
Focus and Motivate:	Suggestions include but not limited to:
Starting Options (Lesson Warm Up)	Find the area of a region under one curve. Then find the area of a region under a second curve underneath that. What is the difference in area?
Teaching Objectives	 Calculate areas in the plan using the definite integral.
Checking for Understanding	Suggestions include but not limited to: Exit Tickets (teacher made supplement) Inquiry Formative Assessment
Practice and Apply: Assigning Homework	Larson Text Section: 6.1 Pg: 416-419

Section: 6.1 – Area of a Region Between Two Curves

Section: 6.2 – Volume: The Disk and Washer Method

	Advanced Placement AB
Focus and Motivate: Starting Options (Lesson Warm Up)	Suggestions include but not limited to: Sketch the general concept behind the Disk Method and its relationship to the area of sircles
Teaching Objectives	Calculate volumes of solids with known cross sections using definite integrals
	 Calculate volumes of solids of revolution using definite integrals.

Checking for Understanding	Suggestions in Exit Tickets Inquiry Formative	nclude but not limi t s (teacher made su Assessment	ted to: pplement)	
Practice and Apply:	Larson Text	Section: 6.2	Pg: 427-430	
Assigning Homework				

PART IV: EVIDENCE OF LEARNING

IDENTIFY THE METHODS BY WHICH STUDENTS WILL DEMONSTRATE THEIR UNDERSTANDING OF CONTENT AND THEIR ABILITY TO APPLY SKILLS. IDENTIFY BLOOM'S LEVELS.



Formative Assessments:

The effectiveness of the instructional program will be based on numerous activities and strategies including the following: teacher observations, students collaborating with peers, questioning strategies, student record-keeping, quizzes, exit/admit assignments, peer/self-assessments, learning/response logs, discussions and practice presentations.

Accommodation(s)/Modification(s): As per Individual Students' IEP or 504 plan.

Summative Assessments:

The following assessments will be used to evaluate student learning, skill acquisition and academic achievement of the Standards of Mathematical Practice and the New Jersey Learning Standards for Mathematics listed under each chapter in the Algebra 1 curriculum/syllabus at the conclusion of an instructional time period.

- Diagnostic Pre-Test
- Chapter Tests
- Projects
- End-Of Course Assessment

Accommodation(s)/Modification(s): As per Individual Students' IEP or 504 plan.

Performance Assessments:

The following assessments require students to utilize various strands of mathematics.

- Projects
- Practice AP Exam Questions
- Homework
- Classwork

PART I: UNIT RATIONALE - UNIT 7

WHY ARE STUDENTS LEARNING THIS CONTENT AND THESE SKILLS?

Unit 7 Title: Integration Techniques, L'Hôpital's Rule and Partial Fractions	Unit Summary : This unit begins with one of the last topics before the AP Exam, L'Hôpital's Rule. It is then followed with a review of all previous topics for the AP Exam itself. Afterwards, if time permits, students will be introduced to	
Grade Level(s): 12	integration by parts and integration of partial fractions.	
AP Topics: 4.7		
 Essential Question(s): How do you evaluate a limit when direct substitution produces an indeterminate form? How do you integrate a complex rational function? What other techniques can be used to evaluate integrals? 	 Enduring Understanding(s): Students will be able to: Determine limits of functions that result in indeterminate forms. Evaluate integrands requiring integration by parts (indefinite only) Evaluate an improper integral or determine that the integral diverges (BC only) 	

PART II: INSTRUCTIONAL STRATEGIES AND RESOURCES

DESCRIBE THE LEARNING TARGETS AND APPLICATIONS OF MATHEMATICAL PRACTICES FOR AP CALCULUS

Mathematical Practices:

MPAC 1 – Implementing Mathematical Processes

Students will apply appropriate mathematical rules or procedures, with and without technology. They will need to complete this with integration with partial fractions and improper integrals.

MPAC 2 – Connecting Representations

Students will identify how mathematical characteristics or properties of functions are related in different representations. This will be achieved using L'Hôpital's Rule and recognizing that different representations of the limit can produce the actual limit of the function.

MPAC 3 – Justification

Students will apply an appropriate mathematical definition, theorem, or test. Students will need to use L'Hôpital's Rule when they realize that the given limit yields an indeterminate form. They will accomplish this using group activities and pair-share work.

MPAC 4 – Communication and Notation

Students will use appropriate mathematical symbols and notation. They will need to show when an integral has been completed but not yet evaluated in the case of definite integrals. This can be achieved in individual practice, pair-share practices and reinforcement in direct instruction.

Real World and Interdisciplinary Problems:

Text: Calculus for AP 2nd Edition by R. Larson & P. Battaglia	Section: 7.7	Page : 515
Description: Velocity in a resisting medium.		
Text: Calculus for AP 2nd Edition by R. Larson & P. Battaglia	Section: 7.5	Page : 499
Description: Epidemic Model.		
Text: Calculus for AP 2nd Edition by R. Larson & P. Battaglia	Section: 7.6	Page : 505
Description: Population - Finding average value of a logistic model.		

Students will Engage with the Following Text, Resources, and Tools:

Text:			
•	Calculus for AP, 2nd Edition by Ron Larson and Paul Battaglia		
Online	e Resources incorporated through the year, include but not limited to:		
•	Albert io – online AP exam practice resource		
•	AP Central - Previously published and released AP questions		
•	CalcChat and Calc View – Online Textbook Resource		
Calculators:			
•	TI – 84 Plus		
•	TI - 89 Titanium		

The Following 21st Century Skills and the 4 Mathematical Practices are Embedded Throughout the Course and are Evident in Daily Lessons, Assignments, Activities, Assessments, and Projects:

21 st Century skills:	NJ Standards for Mathematical Practices:
 Critical thinking Creativity Collaboration Communication Information literacy Technology literacy Media literacy Flexibility Leadership Initiative Productivity Social skills 	 Make sense of problems and persevere in solving them Reason abstractly and quantitatively Construct viable arguments and critique the reasoning of others. Model with mathematics Use appropriate tools strategically Attend to precision Look for and make use of structure Look for and express regularity in repeated reasoning

Students Will Write:

When is L'Hôpital's Rule required?

What happens if L'Hôpital's Rule yields an indeterminate limit the first time it is used?

How can integrals be simplified to be able to use techniques we know?

How do you integrate a complex rational function?

Explain the difference between partial fractions and integration by parts.

PART III: TRANSFER OF KNOWLEDGE AND SKILLS

DESCRIBE THE LEARNING EXPERIENCE.

How Students Uncover Content and Build Skills:

Students will uncover and build skills through various classroom learning activities. Investigating algebra activities, modeling examples, using real-life application, using note-taking strategies, and using Smart Board technologies will all be explored as a blend of learning strategies to promote critical thinking, problem solving and performance skills of all learners. Other learning experiences could include alternative lesson openers, math and history applications, problem-solving workshops, interdisciplinary applications and projects.

Suggested warm-up activities, instructional strategies/activities, and assignments:

Section: 7.7 - Indeterminate Forms and L'Hôpital's Rule

	Advanced Placement AB	
Focus and Motivate:	Suggestions include but not limited to:	
Starting Options (Lesson Warm Up)	Find the limit on the board using techniques that have already been used (factor and canceling, trigonometric identities)	
Teaching Objectives	 Determine limits of functions that result in indeterminate forms. 	
Checking for Understanding	Suggestions include but not limited to: Exit Tickets (teacher made supplement) Inquiry Formative Assessment	
Practice and Apply: Assigning Homework	Larson Text Section: 7.7 Pg: 513-516	

THESE NEXT TWO SECTIONS SHOULD ONLY BE COVERED AFTER THE AP EXAM IF TIME PERMITS

Section: 7.2 – Integration by Parts

	Advanced Placement AB
Focus and Motivate:	Suggestions include but not limited to:
Starting Options (Lesson Warm Up)	Write down the integration by parts formula. What do
	you notice about it? Discussion about how it is set up.

Teaching Objectives	 For integrands requiring integration by parts: Determine indefinite integrals (BC only) Evaluate definite integrals (BC only) 	
Checking for Understanding	Suggestions include but not limited to: Exit Tickets (teacher made supplement) Inquiry Formative Assessment	
Practice and Apply: Assigning Homework	Larson Text Section: 7.2 Pg: 469-472	

Section: 7.5 – Partial Fractions

	Advanced Placement AB
Focus and Motivate:	Suggestions include but not limited to:
Starting Options (Lesson Warm Up)	How do we decompose a function? Go through the process of factoring and finding the basic equation and solving for the variables.
Teaching Objectives	• Evaluate an improper integral or determine that the integral diverges (BC only)
Checking for Understanding	Suggestions include but not limited to: Exit Tickets (teacher made supplement) Inquiry Formative Assessment
Practice and Apply: Assigning Homework	Larson Text Section: 7.6 Pg: 504-505

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